Exam Program Correctness, May 6th 2015, 18:30-21:30h.

- This exam consists of three problems. Problem 1 is worth 20 points, problem 2 is worth 30 points, and problem 3 is worth 40 points. You get 10 points for not misspelling your name and student number.
- Give complete annotations, and linear proofs. Use a pen. Do not use a pencil!
- The exam is a closed book exam. You are not allowed to use the reader, slides, notes, or any other material.
- Do not hand in scratch paper!

Problem 1 ($2 \times 10 = 20$ pt). Declared is the variable $x : \mathbb{Z}$.
(a) Design an annotated command S_0 that satisfies the Hoare triple:

$$\{\ x = X\ \}\ \ S_0\ \ \{\ x > 0\ \wedge\ (x = 2 \cdot X + 1\ \vee\ x = -2 \cdot X)\ \}$$

(b) Design an annotated command S_1 that satisfies the reverse Hoare triple:

$$\{\ x > 0\ \wedge\ (x = 2 \cdot X + 1\ \vee\ x = -2 \cdot X)\ \}\ \ S_1\ \ \{\ x = X\ \}$$

Problem 2 (30 pt). Design and prove the correctness of a command T that satisfies

$$
\begin{array}{l}
\textbf{const}\ n : \mathbb{N},\quad a : \ \textbf{array}\ [0..n)\ \ \textbf{of}\ \mathbb{Z};\\
\textbf{var}\ s : \mathbb{Z};\\
\quad \{\ P :\ \textbf{true}\ \ \}\\
T\\
\quad \{\ Q : s = \Sigma(\Sigma(a[j] \cdot a[k] \mid j, k : i \leq j \leq k < n) \mid i : 0 \leq i < n)\ \}\ .
\end{array}
$$

The time complexity of the command T must be linear in n. Start by defining (a) suitable helper function(s) and the corresponding recurrence(s).

Problem 3 (40 pt). Given is a two-dimensional array a that is *ascending* in its first argument and *decreasing* in its second argument. Consider the following specification:

$$
\begin{array}{l}
\textbf{const}\ n,\ w : \mathbb{N},\quad a : \ \textbf{array}\ [0..n)\ \ \textbf{of}\ \mathbb{N};\\
\quad \textbf{var}\ z : \mathbb{N};\\
\qquad \{P :\ Z = \#\{(i,j) \mid i, j : 0 \leq i\ \wedge\ 0 \leq j\ \wedge\ i^2 + j^2 \leq n^2\ \wedge\ a[i,j] = w\}\ \}\\
\quad U\\
\qquad \{Q :\ Z = z\}
\end{array}
$$

(a) Make a sketch in which you clearly indicate where the array values are high, where they are low, and how a contour line runs.

(b) Define a function $F(x, y)$ that can be used to compute Z. Determine the relevant recurrences for $F(x, y)$, including the base cases.

(c) Design a command U that has a linear time complexity in n. Prove the correctness of your solution. [Note: you can only receive points for part (c) if the recurrences in part (b) are correct.]